

**E 2212**

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Reg. No.....

Name.....

**B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2018**

**Second Semester**

Complementary Course—Statistics

**THEORY OF RANDOM VARIABLES**

(Common for Physics, Mathematics and Computer Applications)

[2013—2016 Admissions]

Time : Three Hours

Maximum Marks : 80

**Part A (Short Answer Questions)**

*Answer all questions.*

*Each question carries 1 mark.*

1. Define random variable.
2. Define probability mass function.
3. Define distribution function of a random variable.
4. Define mathematical expectation.
5. Define characteristic function.
6. Define  $r^{\text{th}}$  raw moment of a random variable.
7. State the measures of kurtosis.
8. What is the principle of least squares ?
9. What do you mean by curve fitting ?
10. Define Karl Pearson's co-efficient correlation.

(10 × 1 = 10)

**Part B (Brief Answer Questions)**

*Answer any eight questions.*

*Each question carries 2 marks.*

11. When do you say a random variable is : (a) Discrete ; and (b) Continuous ?
12. Let  $X$  be a random variable whose p.d.f is constant on an interval  $a \leq X \leq b$  and zero elsewhere  
Find the constant.

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13. Define joint probability density function and state its properties.
14. Establish the relationship between raw moments and central moments.
15. The random variable  $X$  has the p.d.f.,  $f(x) = xe^{-x}$ ,  $x > 0$  and 0 elsewhere. Determine the m.g.f. of  $X$ .
16. If  $M(t)$  is the m.g.f. of a r.v.  $X$ . Explain how to obtain the moments of  $X$  from  $M(t)$ .
17. Interm of moments explain the measures of skewness and kurtosis.
18. What do you mean by absolute moments?
19. How will you fit a straight line to the given data by the method of least squares?
20. What is a scatter diagram? What conclusion can be drawn from it?
21. Show that correlation co-efficient lies between  $-1$  and  $+1$ .
22. Write down the two regression equations.

 $(8 \times 2 = 16)$ 

### Part C (Descriptive / Short Essay Questions)

Answer any six questions.

Each question carries 4 marks.

23. A random variable  $X$  has the following probability density function :

$X$	:	$-2$	$-1$	$0$	$1$	$2$	$3$
$P(X = x)$	:	$0.1$	$0.1$	$a$	$0.2$	$b$	$0.2$

Find  $a$  and  $b$  if  $p(X \leq 0) = 5p(x = 2)$ .

24. Let  $X$  be a continuous random variable with p.d.f.  $f(x)$ . Let  $Y = X^2$ . Find the p.d.f. and distribution function of  $Y$ .
25. A random variable  $X$  has the density function :

$$f(x) = k \frac{1}{1+x^2} \text{ if } -\infty < x < \infty$$

$= 0, \text{ elsewhere}$

Determine  $k$  and the distribution function of  $X$ .

26. A coin is tossed until a head appears. What is the expectation of the number of tosses required?

27. If p.d.f. of the random variable X is  $f(x) = e^{-|x|}$ ,  $-\infty < x < \infty$ . Derive the moment generating function of X.
28. Let the m.g.f. of a random variable be  $M(t) = e^2(t + t^2)$ . Find the first, second and third raw moments.
29. If X has p.d.f.  $f(x) = q^{x-1} p$ ,  $x = 1, 2, 3, \dots$ ,  $0 < p < 1$  and  $q = 1 - p$ . Determine  $E(X)$  and  $V(X)$ .
30. Write a note on scatter diagram.
31. Calculate Karl Pearson correlation coefficient for the following data :

$x :$	6	2	10	4	8
$y :$	9	11	5	8	7

(6 × 4 = 24)

#### Part D (Long Essay Type Questions)

*Answer any two questions.  
Each question carries 15 marks.*

32. If X and Y has the joint density function :

$$f(x, y) = \frac{1}{8}(6 - x - y), 0 < x < 2, 2 < y < 4$$

= 0 otherwise.

- (a) Determine the marginal densities.
- (b) Test whether X and Y are independent.
- (c) Compute  $p(X < 1, Y < 3)$ .
33. Derive Spearman's formula for rank correlation coefficient.
34. Fit a curve of the form  $y = ab^x$  to the following data :

$x :$	1	2	3	4	5	6	7	8
$y :$	1.0	1.2	1.8	2.5	3.6	4.7	6.6	9.1

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35. Consider the following probability distribution :

X	:	-3	-2	-1	0	1	2	3
P (X)	:	0.5	0.1	0.3	0	0.3	0.15	0.1

Find the following :—

(a)  $E(X)$ .

(b)  $E(2X + 3)$

(c)  $V(X)$ .

(2 × 15 = 30)