



21101238

QP CODE: 21101238

Reg No :

Name :

B.Sc DEGREE (CBCS) EXAMINATION, APRIL 2021**Sixth Semester****CORE - MM6CRT02 - GRAPH THEORY AND METRIC SPACES**

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

9FAF2725

Time: 3 Hours

Max. Marks : 80

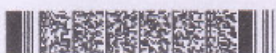
Part A*Answer any ten questions.**Each question carries 2 marks.*

1. Define a Graph. Define a loop in a graph.
2. When will you say that two graphs are isomorphic?
3. Draw all non-isomorphic complete bipartite graphs with atmost 4 vertices.
4. Define a walk. When will you say that a walk is open?
5. Define a tree. Draw a tree which is a complete graph.
6. Define spanning trees. How many spanning trees are there for K_4 ?
7. Define Eulerian graph. Is K_3 Eulerian? Justify.
8. Define closure of a graph . Draw one example.
9. Define metric space.
10. Let (X,d) be a metric space and $A \subseteq X$. Define an interior point of A.
11. Define convergence in a metric space using metric.
12. Define isometry.

(10×2=20)

Part B*Answer any six questions.**Each question carries 5 marks.*

13. Let G be a simple graph with n vertices and let \bar{G} be its complement. Prove that for each vertex v in G , $d_G(v) + d_{\bar{G}}(v) = n-1$.
14. Define incidence matrix of a graph. What can you say about the sum of the elements in the i^{th} row of of the incidence matrix of the graph. Write down the incidence matrix of K_4





15. If G be a graph with n vertices and q edges. Let $\omega(G)$ denote the number of connected components of G . Then prove that G has at least $n - \omega(G)$ edges.
16. a) Define cut vertex of a graph.
b) Let v be a vertex of the connected graph G . Then prove that ' v ' is cut vertex of G if and only if there are two vertices ' u ' and ' w ' of G , both different from ' v ', such that ' v ' is on every $u - w$ path in G .
17. Prove that a simple graph G is Hamiltonian if and only if its closure $C(G)$ is Hamiltonian.
18. Prove that in any metric space X , the empty set \emptyset and the full space X are open sets.
19. Define Cantor set. Prove that there exist infinitely many points in Cantor set.
20. Let X be a metric space. If $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_n \rightarrow x$ and $y_n \rightarrow y$, show that $d(x_n, y_n) \rightarrow d(x, y)$.
21. If $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X , then prove that there exists a point in X which is not in any of the A_n 's.

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22.
 - (a) State and prove First theorem of graph theory.
 - (b) Prove that in any graph G there is an even number of odd vertices.
 - (c) Let G be a k -regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k .
23. a) Let G be simple graph with at least three vertices. Then prove that G is 2- connected if and only if for each pair of distinct vertices u and v of G , there are two internally disjoint $u - v$ paths in G .
b) Let u and v be two vertices of the 2- connected graph. Then prove that there is a cycle passing through both u and v .
24. a) In any metric space X prove that the empty set \emptyset and the full set X are closed sets.
b) Prove that a subset F of a metric space X is closed if and only if its complement F^c is open.
25. a) Prove that if a convergent sequence in a metric space has infinitely many distinct points, then its limit is a limit point of the set of points of the sequence.
b) Will the result be true, if the condition infinitely many distinct points is not given? Justify.

(2×15=30)

