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QP CODE: 22101058

Reg No :

Name :

**B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS,
APRIL 2022**

Sixth Semester

CORE - MM6CRT02 - GRAPH THEORY AND METRIC SPACES

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

DD4116B4

Time: 3 Hours

Max. Marks : 80

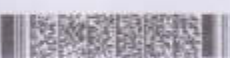
Part A

Answer any ten questions.

Each question carries 2 marks.

1. Define a Graph. When will you say that a graph is simple?
2. Give two different drawings of K_4 which are isomorphic.
3. Define complete bipartite graph. Give an example of a complete bipartite graph which is complete.
4. Define an edge deleted subgraph.
5. Define a spanning tree of a graph G . Draw any two isomorphic spanning trees of K_4 .
6. Define vertex connectivity of a graph. Draw a graph whose vertex connectivity is two.
7. Define a tour and an Euler tour of a graph G .
8. Define Hamiltonian graph, Draw a graph with Hamiltonian path but no Hamiltonian Cycle.
9. Define an open sphere in a metric space X . Give an example.
10. Define closed set in a metric space (X, d) .
11. Define convergence of a sequence in a metric space.
12. Define decreasing sequence of sets in a metric space.

(10×2=20)





Part B

Answer any six questions.

Each question carries 5 marks.

13. State and prove First Theorem of Graph Theory.
14. Define adjacency matrix of a graph. Find the graph whose adjacency matrix is $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. What can you say about the graph if all the entries of the main diagonal are zero?
15. Let T be a tree with at least two vertices and let $P = u_0 u_1 \dots u_n$ be a longest path in T . Then prove that both u_0 and u_n have degree one.
16. Let G be graph with n vertices, where $n \geq 2$. Then prove that any connected graph G has at least two vertices which are not cut vertices.
17. If G is a simple graph with n vertices, where $n \geq 3$, and the degree $d(v) \geq \frac{n}{2}$ for every vertex v of G , Then prove that G is Hamiltonian.
18. Prove that $\text{int } A$ is the union of all open balls in A .
19. Define Cantor set. Prove that there exist infinitely many points in Cantor set.
20. State and prove Cantor's intersection Theorem.
21. If a complete metric space is the union of a sequence of its subsets, then prove that the closure of at least one set in the sequence must have non-empty interior.

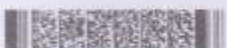
(6×5=30)

Part C

Answer any two questions.

Each question carries 15 marks.

22. State and prove the necessary and sufficient condition for a nonempty graph with atleast two vertices to be bipartite.
23. a) Let 'e' be an edge of the graph G and let ' $G - e$ ' be the sub graph obtained by deleting e . Then prove that $\omega(G) \leq \omega(G - e) \leq \omega(G) + 1$.
b) If G be a graph with n vertices and q edges. Let $\omega(G)$ denote the number of connected components of G . Then prove that G has at least $n - \omega(G)$ edges.





24. Let X be the collection of all bounded real valued functions on $[0, 1]$. Prove that d_1 and d_2 defined below are metrics in X .
- a) $d_1(x, y) = \|f - g\|$, where $\|f\| = \sup\{|f(x)| : x \in [0, 1]\}$.
- b) $d_2(x, y) = \|f - g\|$, where $\|f\| = \int_0^1 |f(x)| dx$
25. Let X be a metric space, let Y be a complete metric space and let A be a dense subspace of X . If f is a uniformly continuous mapping of A into Y , then prove that f can be extended uniquely to a uniformly continuous mapping g of X into Y .

(2×15=30)

