22101058

QP CODE: 22101058

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, APRIL 2022

Sixth Semester

CORE - MM6CRT02 - GRAPH THEORY AND METRIC SPACES

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

DD4116B4

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define a Graph. When will you say that a graph is simple?
2. Give two different drawings of K4 which are isomorphic.
3. Define complete bipartite graph.Give an example of a complete bipartite graph which is complete.
4. Define an edge deleted subgraph.
5. Define a spanning tree of a graph G. Draw any two isomorphic spanning trees of K4 .
6. Define vertex connectivity of a graph. Draw a graph whose vertex connectivity is two.
7. Define a tour and an Euler tour of a graph G.
8. Define Hamiltonian graph, Draw a graph with Hamiltonian path but no Hamiltonian Cycle.
9. Define an open sphere in a metric space X. Give an example.
10. Define closed set in a metric space (X,d).
11. Define convergence of a sequence in a metric space.
12. Define decreasing sequence of sets in a metric space.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. State and prove First Theorem of Graph Theory.

14. Define adjacency matrix of a graph. Find the graph whose adjacency matrix is $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. What can you say about the graph if all the entries of the main diagonal are zero?

15. Let T be a tree with at least two vertices and let $P = u_0u_1 \ldots\ldots\ldots\ldots u_n$ be a longest path in T. Then prove that both u_0 and u_n have degree one.

16. Let G be graph with n vertices, where $n \geq 2$. Then prove that any connected graph G has at least two vertices which are not cut vertices.

17. If G is a simple graph with n vertices, where $n \geq 3$, and the degree $d(v) \geq \frac{n}{2}$ for every vertex v of G, Then prove that G is Hamiltonian.

18. Prove that int A is the union of all open balls in A.

19. Define Cantor set. Prove that there exist infinitely many points in Cantor set.

20. State and prove Cantor's intersection Theorem.

21. If a complete metric space is the union of a sequence of its subsets, then prove that the closure of at least one set in the sequence must have non-empty interior.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. State and prove the necessary and sufficient condition for a nonempty graph with atleast two vertices to be bipartite.

23. a) Let 'e' be an edge of the graph G and let 'G – e' be the sub graph obtained by deleting e. Then prove that $\omega(G) \leq \omega(G-e) \leq \omega(G)+1$.
b) If G be a graph with n vertices and q edges. Let $\omega(G)$ denote the number of connected components of G. Then prove that G has at least $n - \omega(G)$ edges.

24. Let X be the collection of all bounded real valued functions on [0,1]. Prove that d_1 and d_2 defined below are metrics in X.

a) $d_1(x,y) = \|f-g\|$, where $\|f\| = \sup\{|f(x)| : x \in [0,1]\}$.

b) $d_2(x,y) = \|f-g\|$, where $\|f\| = \int_0^1 |f(x)|dx$

25. Let X be a metric space, let Y be a complete metric space and let A be a dense subspace of X. If f is a uniformly continuous mapping of A into Y,then prove that f can be extended uniquely to a uniformly continuous mapping g of X into Y.

(2×15=30)