24001055

QP CODE: 24001055

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, MARCH 2024

Sixth Semester

CORE COURSE - MM6CRT02 - GRAPH THEORY AND METRIC SPACES

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

4F7032EF

Time: 3 Hours Max. Marks : 80

Part A

Answer any ten questions.

*Each question carries **2** marks.*

1. Define neighbourhood set of a vertex in a graph.
2. Give two different drawings of K3,3 which are isomorphic.
3. Define a complete bipartite graph. Give an example.
4. Define supergraph of a graph .
5. Define an acyclic graph. Draw any four non - isomorphic trees with 6 vertices.
6. State Cayley's formula for number of spanning trees of a complete graph. How many spanning trees are there for K3?
7. Define vertex connectivity of a graph. Draw a graph whose vertex connectivity is one.
8. Define a maximal non Hamiltonian graph. Give an example.
9. Prove that in any metric space X, the full space X is open.
10. Define boundary of a set in a metric space X.
11. Define limit of a sequence in a metric space.
12. When do we say that two metric spaces X and Y are isometric?

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

Turn Over

13. Let G be a graph with n vertices and e edges and let m be the smallest positive integer such that $m \geq 2e/n$. Prove that G has vertex of degree atleast m.

14. Define adjacency matrix of a graph. Draw the graph whose adjacency matrix is

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}.$$

What can you say about the graph if all the entries of the main diagonal are zero?

15. Let G be graph with n vertices , where $n \geq 2$. Then prove that any connected graph G has at least two vertices which are not cut vertices.

16. A connected graph G has an Euler trail if and only if it has at most two odd vertices.

17. If G is a simple graph with n vertices , where n ≥ 3 , and the degree $d(v) \geq \frac{n}{2}$ for every vertex v of G, Then prove that G is Hamiltonian.

18. Prove that int A is the union of all open balls in A.

19. Write a short note on boundary of a set.

20. Is limit of a sequence, a limit point of the underlying set? Justify with suitable examples.

21. State and prove Cantor's intersection Theorem.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a)Let G be a nonempty graph with atleast two vertices. Prove that if G is bipartite then it has no odd cycles.
(b) Is the converse true? Justify your answer.

23. a) Let 'e' be an edge of the graph G and let 'G – e' be the sub graph obtained by deleting e. Then prove that $\omega(G) \leq \omega(G-e) \leq \omega(G)+1$.
b) If G be a graph with n vertices and q edges. Let ω(G) denote the number of connected components of G . Then prove that G has at least n –ω(G) edges.

24. a) Define metric space with example.
b) Let (X,d) be a metric space. Show that d*, defined by d*(x,y)= $\frac{d(x,y)}{1+d(x,y)}$ is also a metric on X.

25. (a) If $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X, then prove that there exists a point in X which is not in any of the $A_n' s$.
(b) State Baire's theorem. Explain how it is related to the above result.

(2×15=30)