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Reg. No................................

Name..................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, FEBRUARY 2018

First Semester

Faculty of Science

Branch I (a) : Mathematics

MT 01 C03—MEASURE THEORY AND INTEGRATION

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Show that translates of measurable sets are measurable.
2. Show that the product of two measurable functions is again measurable.
3. If f is integrable show that $|f|$ is integrable. Is the converse true ? Justify.
4. State the bounded convergence theorem.
5. State Lebesgue dominated convergence theorem.
6. Define Radon-Nikodym derivative of v with respect to μ.
7. Define convergence in measure with an example.
8. Define measurable rectangles. Show that the representation of a rectangle in the form $A \times B$ need not be unique.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Prove that the sum and product of two simple functions are simple.
10. Suppose f is a measurable real-valued function and g a continuous function defined on $(-\infty, \infty)$. Show that $g \circ f$ is measurable.

Turn over

11. Let $< f_n >$ be a seqence of integrable functions such that $f_n \to f$ a.e. with f integrable. Prove that $\int |f - f_n| \to 0$ if and only if $\int |f_n| \to \int |f|$.

12. Show that Monotone convergence theorem need not hold for decreasing sequences of functions.

13. Show that if υ is a signed measure such that $\upsilon \perp \mu$ and $\upsilon << \mu$, then $\upsilon = 0$.

14. Show that the Radon-Nikodym derivative has the property $\left[\frac{d\upsilon}{d\lambda}\right] = \left[\frac{d\upsilon}{d\mu}\right]\left[\frac{d\mu}{d\lambda}\right]$, if $\upsilon << \mu << \lambda$.

15. Show that if $f_n \to f$ in measure and $g_n \to g$ in measure, then $f_n + g_n \to f + g$ in measure.

16. If $f_n \to f$ a.u then prove that $f_n \to f$ a.e.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Show that a function which is almost everywhere equal to a measurable function is measurable.

18. State and prove Monotone convergence theorem.

19. (a) Let f be a non-negative integrable function. Show that the function $F(x) = \int_{-\infty}^{x} f$ is continuous.

    (b) Suppose f is a bounded function defined on $[a, b]$. Prove that f is measurable and $R\int_a^b f(x)\,dx = \int_a^b f(x)\,dx$. if f is Riemann Integrable.

20. State and prove Radon-Nikodym theorem.

21. Let $f_n \to f$ a.e. Suppose that $|f_n| \le g$ (an integrable function) prove that $f_n \to f$ a.u.

22. State and prove Fubini's theorem.

(3 × 5 = 15)